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Transient and crisis-induced intermittencies in high-power ferromagnetic resonance

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Pulsed ferromagnetic resonance experiments at unusually high power levels reveal the existence of long-lived transient forms of type-III Pomeau-Manneville intermittency and crisis-induced bursting in a spin-wave system. The average lifetime of this particular chaotic transient as well as the mean duration between chaotic bursts are shown to obey a scaling law. The results are in good qualitative agreement with numerical simulations based on a two-mode spin-wave model.

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I. INTRODUCTION

Several branches of science have benefited by the progress in the theory of nonlinear dynamical systems that occurred over the last decade. Nowadays three universal routes to chaos are well established, namely, period doubling, intermittency [Pomeau-Manneville (PM)], and quasiperiodicity [1]. In recent years, considerable effort has been focused on regimes beyond the onset of chaos, the next step toward a better understanding of the transition to turbulence. Spatiotemporal patterns and sudden changes that strange attractors can undergo as a system parameter is varied are examples of subjects of current interest. For dissipative systems, a formal theory describing sudden discontinuous changes in a chaotic attractor was introduced by Grebogi, Ott, Yorke (GOY) *et al.* several years ago [2]. They have termed such changes "crises," which can be of three types: *attractor destruction, attractor widening,* and *attractor merging*. In all cases, a time scale that quantifies the postcrisis behavior has been predicted. For attractor destruction, where the characteristic behavior is the existence of transient chaos, the time scale is the chaotic transient lifetime averaged on a number of different initial conditions. As the control parameter p is varied through its critical value p_c, the mean duration $\langle T \rangle$ is predicted to scale as $|p-p_c|^{-\gamma}$, where γ is the crisis critical exponent. The first experimental report of this scaling law for chaotic transients was provided by Carroll, Pecora, and Rachford [3] in a spin-wave experiment. At driving frequencies between 2.0 and 3.4 GHz, their experimental findings suggested the existence of "multiple attractors," while numerical simulations suggested the need of more than three interacting spin-wave modes to explain the long-lived transients they had observed in the experiments. Surprisingly, very little has been said about transient chaos in other dynamical systems. On the other hand, only recently experimental confirmations of the scaling theory for attractor merging and attractor widening have been observed in a magnetically driven mechanical system [4], a laser [5], and a convecting superfluid [6]. The purpose of this paper is twofold: First, we report on the observation of transient forms of type-III PM intermittency. This transition is associated with a subcritical period-doubling bifurcation and differs from the usual chaotic transient considered by GOY, which is associated with a collision of a strange attractor and its unstable manifold. However, it is shown that there is a scaling law governing the transient intermittency, similarly to the GOY scenario. For another set of experimental parameters, we then present the first evidence of chaotic bursting (attractor widening) in high-power ferromagnetic resonance (HPFMR). Contrary to a previous belief [3], we show that a two-mode model accounts satisfactorily for the experimental observations.

II. EXPERIMENTS

The experimental arrangement is sketched in the inset of Fig. 1. It consists of a sphere (S) of ferrimagnetic yttrium iron garnet (YIG) with diameter 1 mm in the center of a rectangular TE_{102} microwave cavity ($Q \sim 3000$), placed between the poles of a magnet (M), which provides the static magnetic field H_0. The frequency ($f_p = 8.9$ GHz) of the microwave generator (G) is stabilized at the resonance of the cavity, which is coupled to a waveguide through a circulator (C). The interaction with the sample results in changes in the amplitude of the

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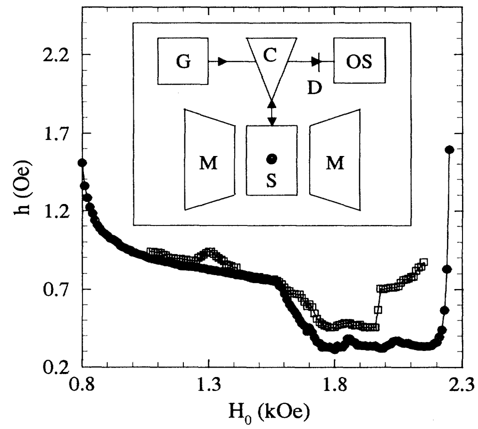

FIG. 1. Measured Suhl instability threshold (solid circles) and spin-wave auto-oscillation threshold (open squares) for H_0 parallel to the [100] crystal axis. Solid lines are guides to the eyes. Inset: Schematic of the experiment, as described in the text.

radiation reflected by the cavity, which are detected by a rectifier diode (D) at the output port of the circulator and stored in an oscilloscope (OS) board connected to a computer. In order to study the transient response, the microwave is pulsed by a *p-i-n* modulator before being amplified by a 10-W traveling wave tube (not shown in Fig. 1). Pulses up to 2 ms long were comfortably used with no detectable heating effects. The experiments are done with fixed H_0 and varying microwave magnetic field h, in the subsidiary resonance configuration [7]. At low-power levels, the pulse reflected from the microwave cavity has essentially the same shape as the incoming microwave pulse. As the power is increased, abrupt changes in the pulse shape occur due to spin-wave instabilities at subsequent thresholds, namely, the Suhl [7] instability threshold h_c (solid circles in Fig. 1), the spin-wave auto-oscillation threshold h_c' (open squares in Fig. 1), and a sequence of bifurcations that lead to spin-wave chaos [8]. Henceforth, we will consider $R \equiv h/h_c$ as our control parameter.

The *destruction* of a type-III PM intermittency has been observed with $H_0 = 1830$ Oe, parallel to the [100] crystal axis. For $R > R_c = 2.88$, we observe an intermittent spin-wave auto-oscillation ($f \sim 1$ MHz) with laminar regions interrupted by chaotic bursts. For $2.82 < R < R_c$, the steady-state response is periodic after a chaotic transient whose duration swings randomly, as one observes on the oscilloscope screen. The upper panel in Fig. 2 shows a frozen digitized version of a chaotic transient with duration $T \sim 618$ μs, with $R = 2.84$. The presence of a subharmonic at $f/2$, characteristic of a type-III PM intermittency, is better seen in the expanded region shown in the lower panel of Fig. 2. Since each microwave pulse finds the spin-wave system in a different state, each transient corresponds to a different initial condition. As in a crisis [2], the length of a chaotic transient for a particular orbit depends sensitively on the initial condition and can be remarkably long. By averaging the transients from 100 pulses at fixed microwave power, we have measured the mean duration $\langle T \rangle$ showed in the log-log plot of Fig. 6(a) (solid circles). The slope of the linear regression (solid line) is -0.503.

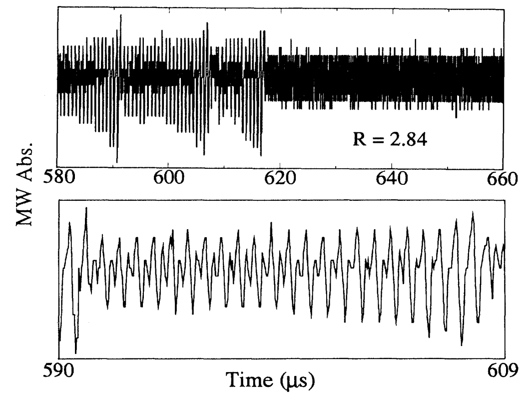

FIG. 2. Upper panel: Detected microwave absorption in the subsidiary resonance in a YIG sphere showing a transient type-III PM intermittency with duration $T \sim 618$ μs at $R = 2.84$. Lower panel: Expansion of a laminar region of the upper panel.

Figure 3 shows the scenario corresponding to the crisis-induced intermittency between a small strange attractor and a larger one, observed with $H_0 = 1950$ Oe, parallel to the [111] crystal direction. We have measured the mean time between bursts from a number of pulsed time series at each value of the control parameter. The closer to $R_c \sim 4.05$, the greater the number of pulses needed to average the duration over 100 measurable residence times in the smaller attractor. The results are shown in Fig. 6(c) (solid circles). From the slope of the linear regression, we estimate a critical exponent $\gamma \sim 0.49$.

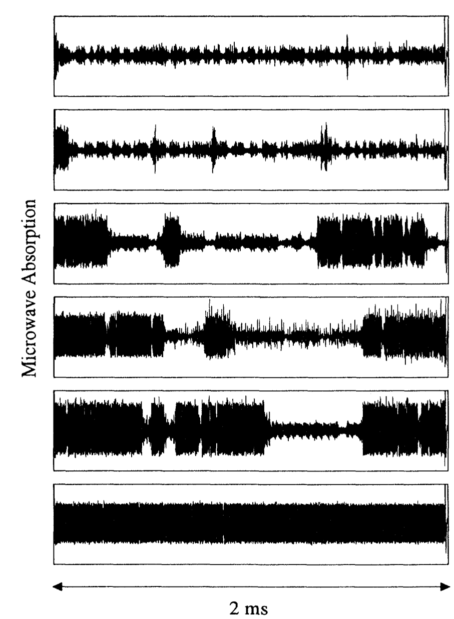

FIG. 3. Observed chaotic-bursting scenario. $R = 4.119$, 4.160, 4.198, 4.241, 4.284, and 4.373, from top to bottom, respectively.

III. NUMERICAL SIMULATIONS

The efforts in modeling the instabilities observed in high-power ferromagnetic resonance experiments are based on a theory introduced by Suhl [7] in the mid 1950s and have been phrased in terms of the excitation and the nonlinear interaction of spin waves in the sample. As in other dynamical systems, the problem of the number and the nature of the excited modes is an important issue that has not yet been settled. However, most experimental observations have been explained by a model involving a few interacting spin-wave modes [8–13]. As an approximation, we consider the dynamics of only two excited modes. The equations of motion for these coupled spin-wave oscillators may be derived semiclassically through the Landau-Lifshitz equation or quantum mechanically through the Heisenberg equation. For only two modes, the equations for the slowly varying spin-wave variables become [9]

$$\dot{c}_1 = -(\gamma_1 + i\Delta\omega_1)c_1 - ih\rho_1[c_1^* + \alpha\exp(i\beta/2)c_2^*] - i2(U_1 c_1^2 c_1^* + U_{12} c_1^* c_2^2 + 2V_{12} c_2 c_2^* c_1) , \quad (1)$$

$$\dot{c}_2 = -(\gamma_2 + i\Delta\omega_2)c_2 - ih\rho_2[c_2^* + \alpha\exp(-i\beta/2)c_1^*] - i2(U_2 c_2^2 c_2^* + U_{12} c_2^* c_1^2 + 2V_{12} c_1 c_1^* c_2) , \quad (2)$$

where γ_i is the relaxation rate, $\Delta\omega_i = \omega_i - \omega_p/2$, ρ_i is the coupling factor between mode i and the pumping field h, the Us and Vs are nonlinear coupling parameters, β is the phase difference between modes 1 and 2, and α is a parameter which depends on the wave-vector mismatch $\Delta k = k_1 - k_2$. Equations (1) and (2) with their 13 independent parameters represent a very complex mathematical problem but can be solved in a computer to give the time evolution of the spin-wave density $n_i = c_i^* c_i$. For $h \gtrsim h_{ci} = (\gamma_i^2 + \Delta\omega_i^2)/\rho_i$, the solutions are attracted to fixed points. For $h > h_{ci}$, the set has a wide variety of solutions including limit cycles and strange attractors, depending on the relative values of the parameters [7,9] Physically this corresponds to an alternating energy shuffling between the two modes, resulting in modulated microwave absorption. The results shown in Fig. 4 were obtained with $\gamma_1 = \gamma_2$, $\Delta\omega_1/\gamma_1 = 0.2$, $\Delta\omega_2/\gamma_1 = -0.5$, $\rho_1 = \rho_2$, $U_1/\gamma_1 = 0.5$, $U_2/\gamma_1 = 0.5$, $U_{12}/\gamma_1 = -0.2$, $V_{12}/\gamma_1 = -0.4$, $\alpha = 0.65$, and $\beta = \pi$. The upper panel in Fig. 4 shows a representative result for the total spin-wave density $N = n_1 + n_2$, where we observe a type-III PM intermittency with $R = 1.850$. In the lower panel of Fig. 4 the intermittent attractor is *destroyed* after a transient $\gamma_1 T \sim 410$ from the initial condition, with $R = 1.855\,99$. For clarity, we show only the local maxima separated by twice the period of the oscillation. Notice that the steady-state orbit is a period-18 oscillation. The frequency in this case is $f \sim \gamma_1$. In YIG, the relaxation rate $\gamma_k \sim 10^6$ s^{-1}, so that the calculated and measured frequencies are of the same order of magnitude. The transient intermittency appears at $R_c = 1.855\,958\,1 \cdots$. By an averaging of 200 randomly chosen initial conditions within a small volume on the strange attractor at each value of the control parameter, we have obtained the results shown in Fig. 6(b) (solid circles). The straight line is a regression with a slope -0.312, in reasonable agreement with the experimental one. Actually, this relatively small exponent indicates that the transient intermittency occurs in a sharp interval of the control parameter [2] and, presumably, we have not reached the asymptotic region close to R_c, where the computation of the transient durations is prohibitively long. In other words, we believe that a better quantitative agreement between the measured and calculated critical exponents would be possible, providing one were able to approach the asymptotic region.

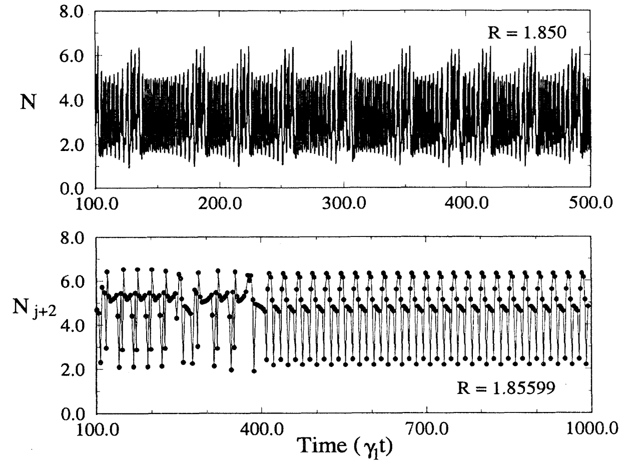

FIG. 4. Calculated transient spin-wave intermittency. Parameters are given in the text.

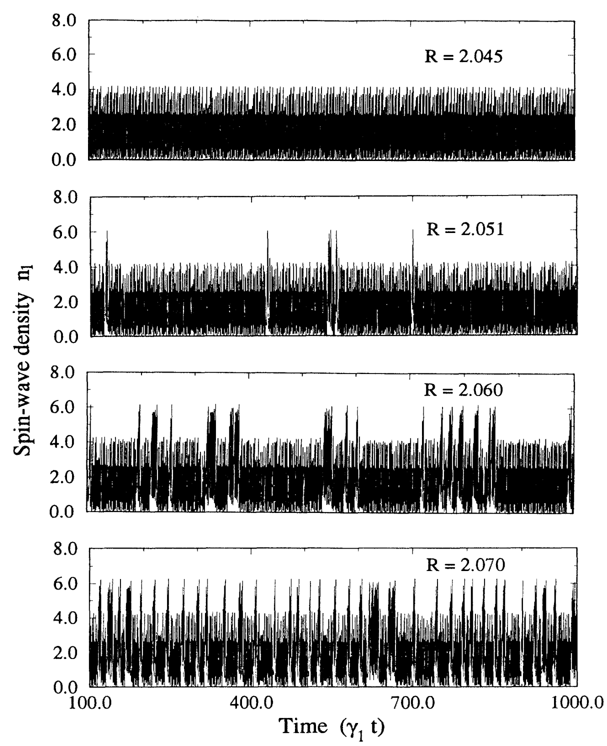

FIG. 5. Calculated chaotic-bursting scenario. Parameters are given in the text.

Figure 5 shows numerical solutions of Eqs. (1) and (2) corresponding to the attractor-widening scenario. They were obtained with $\gamma_1=\gamma_2$, $\Delta\omega/\gamma_1=0.2$, $\Delta\omega_2/\gamma_1=-0.2$, $\rho_1=\rho_2$, $U_1/\gamma_1=0.5$, $U_2/\gamma_1=0.5$, $U_{12}/\gamma_1=-0.4$, $V_{12}/\gamma_1=-0.4$, $\alpha=0.65$, and $\beta=\pi$. In this case, crisis occurs at $R_c=2.047\,14\cdots$. At each value of the control parameter, time series long enough to observe 100 episodes were used to calculate the mean time between bursts. In Fig. 6(d), we show the log-log plot for this transition. As in the experiments, we estimate a critical exponent $\gamma\sim0.5$. Although this value might be seen as a realization of the trivial case [2], we would like to point out that no period-doubling sequence has been observed prior to the chaotic bursting scenario shown in Fig. 3, which occurs well above the Suhl instability threshold.

In summary, we present results of HPFMR, which show that transient times of type-III PM intermittency obey a scaling law, similarly to crisis-induced chaotic transients. To our knowledge, this is the first report on transient times for intermittency. We believe that our results might stimulate nonlinear dynamicists to develop a theory on critical exponents of transient intermittency. In addition, we report the observation of crisis-induced intermittency in HPFMR, also governed by a scaling law, as predicted by GOY. Numerical simulations based on a two-mode model show intriguing similarities with the experiments. We are still in need of results that could shed light on the nature of the excited spin-wave modes beyond the Shul threshold. Experiments that might point in this direction are underway.

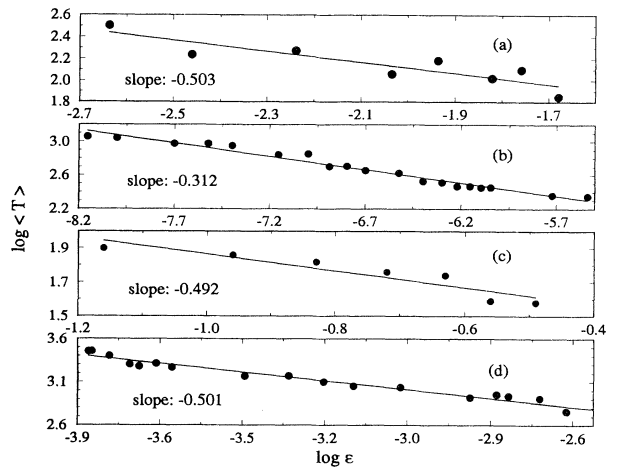

FIG. 6. Log-log plots used to estimate the critical exponents, as described in the text. Here $\varepsilon=|R-R_c|$.

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